

*Tutorial*

# Mohr on Receiver Noise

Characterization, Insights & Surprises

**Richard J. Mohr, PE**

President, R.J. Mohr Associates, Inc.

*Presented to the*

Microwave Theory & Techniques Society

*of the*

IEEE Long Island Section

[www.IEEE.LI](http://www.IEEE.LI)

# Purpose

The purpose of this presentation is to describe the background and application of the concepts of Noise Figure and Noise Temperature for characterizing the fundamental limitations on the absolute sensitivity of receivers\*.

*\* “Receivers” as used here, is general in sense and the concepts are equally applicable to the individual components in the receiver cascade, both active and passive, as well as to the entire receiver. These would then include, for example, amplifier stages, mixers, filters, attenuators, circuit elements, and other incidental elements.*

# Approach

- A step-by-step approach is taken to establish the basis, and concepts for the absolute characterization of the sensitivity of receivers, and to provide the foundation for a solid understanding and working knowledge of the subject.
- Signal quality in a system, as characterized by the signal-to-noise power ratio (S/N), is introduced, but shown to not be a unique characterization of the receiver alone.
- The origin of the major components of receiver noise and their characteristics are then summarized
- Noise Figure,  $F$ , is introduced which uniquely characterizes the degradation of S/N in a receiver.
- The precise definitions of Noise Figure and of its component parts are presented and illustrated with models and examples to provide valuable insight into the concepts and applications.
- The formulation for the Noise Figure of a cascade of devices is derived and illustrated by examples.

# Approach (Cont'd)

- The concept of Noise Temperature,  $T_e$ , is introduced and is shown to be directly derivable from Noise Figure.
- $T_e$  is shown to be a more concise characterization of the receiver alone, by completely eliminating source noise from the equation.
- Its application is illustrated by examples.
- The basic methods of measurement of Noise Figure and Noise Temperature are described and compared.
- Finally, a Summary reviews the material presented, and recommendations are provided for further study.
- References are provided
- An annotated bibliography follows that is intended to serve as a basis for further study

# Introduction

## Signal/Noise (S/N) Characterization

- Introduction of vacuum tube amplifier
  - Discovered there are limits on achievable signal sensitivity
- Achievable signal sensitivity in a Communication, Radar, or EW receiving system is always limited ambient noise along with the output signal
- Signal quality is characterized by the final output Signal/Noise (S/N) ratio
  - Depending on application, S/N of at least 10 or 20 dB, or more, may be required
  - S/N at the input of a receiver is the best it will be
  - Each component in the receiver cascade, while performing its intended function, degrades the output S/N

# Introduction (Cont'd)

## Factors Affecting System S/N

- In a communication system, S/N is a function of:
  - Transmitter output power
  - Gain of Transmit and Receive antennas
  - Path loss
  - Receiver noise - the topic of this presentation
- To characterize the receiver alone, Friis<sup>(1)</sup> introduced Noise Figure which characterized the degradation in S/N by the receiver.
  - Noise Figure of a receiver is the ratio of the S/N at its input to the S/N at its output

# Receiver Noise Summary (Cont'd)

## Thermal Noise<sup>(2,3)</sup>

- Thermal noise (Johnson Noise) exists in all resistors and results from the thermal agitation of free electrons therein
  - The noise is white noise (flat with frequency)
  - The power level of the noise is directly proportional to the absolute temperature of the resistor
  - The level is precisely  $e_n^2 = 4kTRB$  ( $V^2$ ), or  $4kTR$  ( $V^2/Hz$ )
    - Where,
      - $k$  is Boltzman's constant  $= 1.38 \times 10^{-23}$  Joules/ $^{\circ}K$
      - $T$  is the absolute temperature of the resistor in  $^{\circ}K$
      - $R$  is the value of the resistance in Ohms
      - $B$  is the effective noise bandwidth
  - The available noise power is  $e_n^2/4R = kTB$
  - At  $T = T_o = 290^{\circ}K$  (the standard for the definition of Noise Figure),  $kT_o B = 4.00 \times 10^{-21}$  W/Hz ( $= -204.0$  dBW/Hz  $= -174.0$  dBm/Hz  $= -114.0$  dBm/MHz)

*Thermal noise in the resistance of the signal source is the fundamental limit on achievable signal sensitivity*

# Receiver Noise Summary (Cont'd)

## Shot Noise<sup>(4,7)</sup>

- Shot Noise was studied by Schottky, who likened it to shot hitting a target
  - Results from the fluctuations in electrical currents, due to the random passage of discrete electrical charges through the potential barriers in vacuum tubes and P-N junctions
  - Its noise characteristic is white
  - The power level of the noise is proportional to the level of the current through the barrier
  - In vacuum tube diodes, in temperature-limited operation, shot noise is precisely,  $i_n^2(f) = 2eI_0(A^2/Hz)$ , where  $I_0$  is the diode current and  $e$  is the electronic charge =  $1.6 \times 10^{-19}$  Coulombs
  - Vacuum tube diodes, in temperature-limited operation, were the first broadband noise sources for measurement of receiver noise figure.



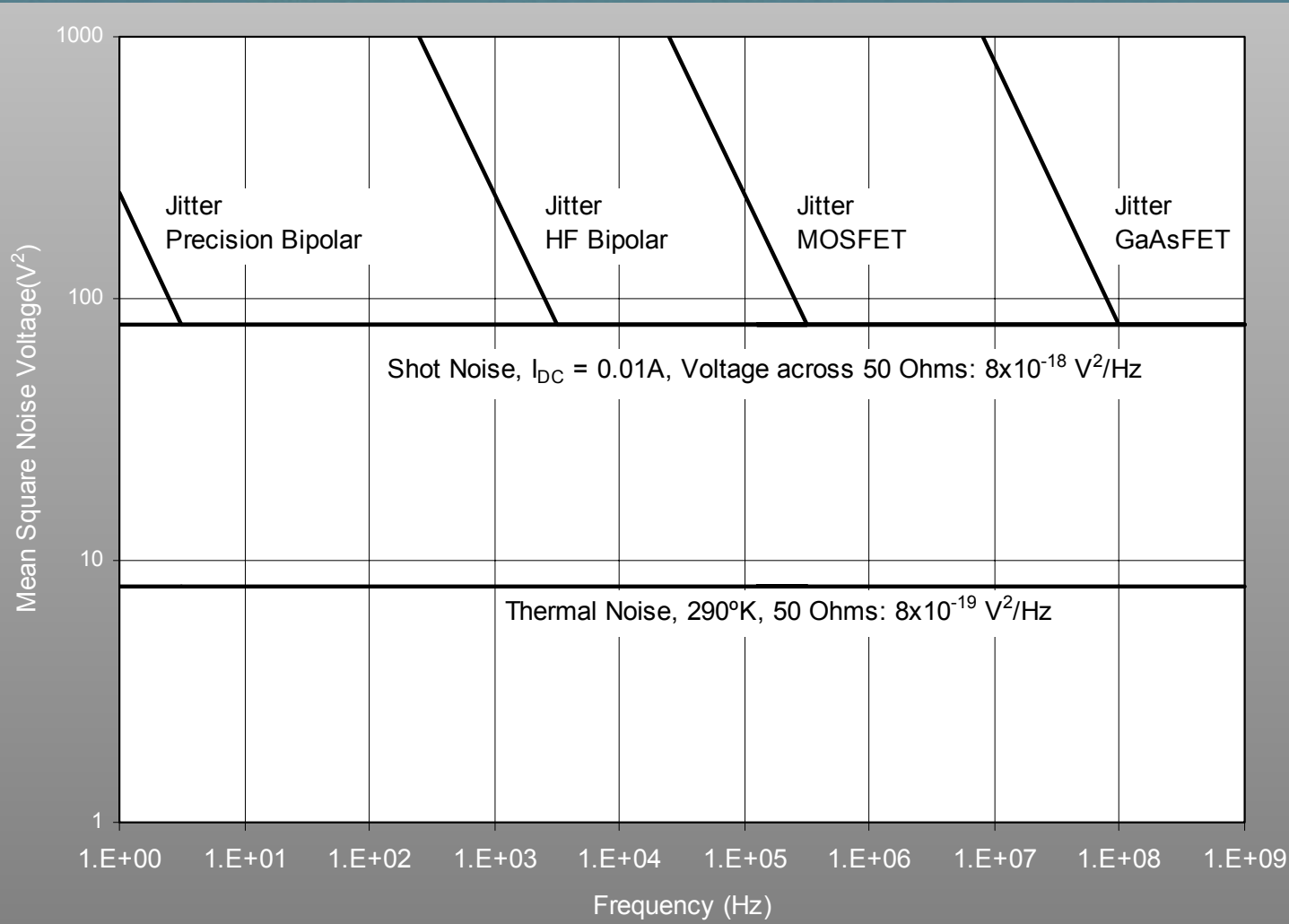
# Receiver Noise Summary (Cont'd)

## Flicker (1/f) Noise<sup>(4,7)</sup>

- Flicker Noise appears in vacuum tubes and semiconductor devices at very low frequencies
  - Its origin is believed to be attributable to contaminants and defects in the crystal structure in semiconductors, and in the oxide coating on the cathode of vacuum tube devices
  - Commonly referred to as 1/f noise because of its low-frequency variation
  - Its spectrum rises above the shot noise level below a corner frequency,  $f_L$ , which is dependent on the type of device and varies from a few Hz for some bipolar devices to 100 MHz for GaAs FETs

# Receiver Noise Summary (Cont'd)

## Comparison of Levels of Major Components of Receiver Noise



# Noise Figure, F

## Definition <sup>\*(1)</sup>

$$F = \frac{S_i / N_i}{S_o / N_o} = \frac{N_o}{GN_i} = \frac{N_o}{GkT_oB} = \frac{GkT_oB + N_R}{GkT_oB}$$

- $F$  is the Noise Figure of the receiver
- $S_i$  is the available signal power at the input
- $S_o$  is the available signal power at the output
- $N_i = kT_oB$  is the available noise power at the input
- $T_o$  is the absolute temperature of the source resistance, 290 °K
- $N_o$  is the available noise power at the output, and includes amplified input noise
- $N_R$  is the noise added by the receiver
- $G$  is the available gain of the receiver
- $B$  is the effective noise bandwidth of the receiver

*\*The unique and very precise definition of Noise Figure and its component parts makes provision for a degree of mismatch between component parts of a receiver chain which is often necessary for minimum Noise Figure*

# Noise Figure

## Example, Relating Noise Figure to Sensitivity

What level of input signal,  $S_i$ , is required for an output  $S_o/N_o = 10$  dB in a receiver with  $NF = 6$  dB, and  $B = 0.1$  MHz?

- From definition of F:

$$F = \frac{N_o}{GkT_oB}$$

and so,

$$N_o = FGkT_oB$$

- Input sensitivity is evaluated by referring the output noise,  $N_o$ , to the receiver's input, i.e.

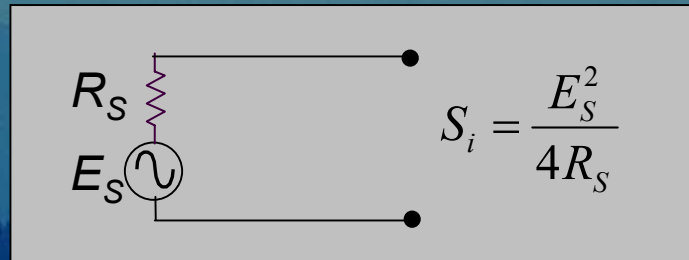
$$N_{oi} = \frac{N_o}{G} = FkT_oB$$

- $N_{oi}(\text{dBm}) = NF(\text{dB}) + KT_o(\text{dBm/MHz}) + 10 \text{ Log } B(\text{MHz}) = 6 - 114 - 10 = -118$  dBm
- For a desired  $S_o/N_o$  of 10 dB,  $S_i$  must be at least:  
 $S_i = -118 \text{ dBm} + 10 \text{ dB} = -108 \text{ dBm}$

# Definition of Factors in Noise Figure

## Available Input Signal Power ( $S_i$ )

- $S_i$  is the signal power that would be extracted from a signal source by a load conjugately\* matched to the output of the source i.e.:

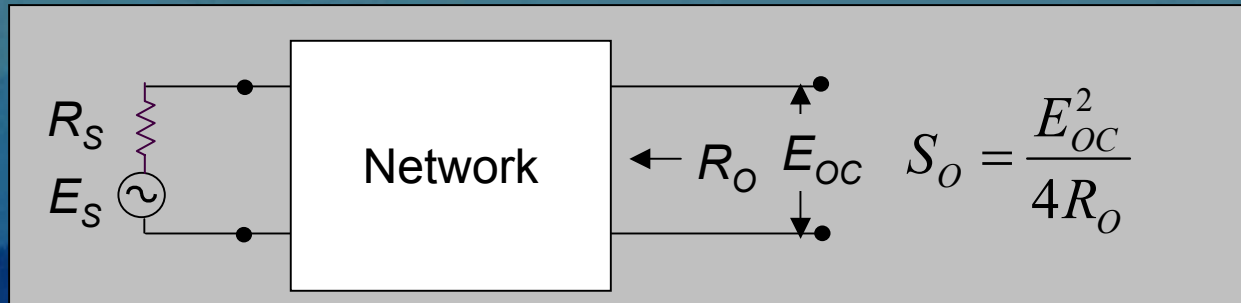


- $S_i$  is dependent only on the characteristics of the source, specifically it is independent of the impedance of the actual load,  $R_L$ .
- For a load,  $R_L \neq R_S$ , the delivered power is less than the available power, but the available power is still  $S_i$ .
- \* *For simplicity and without loss of illustrative value, ideal transformers and reactive elements are not included in models here since they are loss-free and do not directly contribute to receiver Noise Figure*

# Definition of Factors in Noise Figure (Cont'd)

## Available Output Signal Power, $S_O$

- $S_O$  is the power that would be extracted by a load conjugately matched to the output of the network, i.e.:



- $S_O$  is dependent only on the characteristics of the network and its signal source, and the impedance match at its input
- $S_O$  is independent of the actual load,  $R_L$ , on the network
  - For a load,  $R_L = R_O$ , the delivered power will equal the available output power,  $S_O$
  - for  $R_L \neq R_O$  the delivered power will be less than the available output power, but the available output power is still  $S_O$
  - Available output is maximum achievable when input is matched to  $R_s$

# Definition of Factors in Noise Figure (Cont'd)

Available Gain\*, G

$$G = S_o / S_i$$

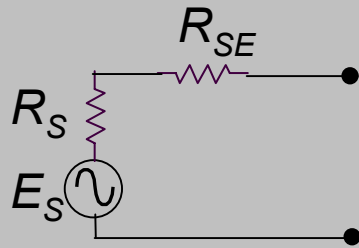
- Definition is applicable to both active and passive devices
- G is independent of impedance match at output
- G is dependent on impedance match at the input
- In general, G is:
  - Less than, or equal to, the maximum available gain
  - Equal to the maximum available gain when source is matched to the input
  - Greater than, or equal to, the insertion gain
  - May be less than, greater than, or equal to, one (unity)

*\*When used herein, G will always refer to the available gain*

# Definition of Factors in Noise Figure (Cont'd)

## Available Gains Of Elementary Networks

### Gain of Series Resistor

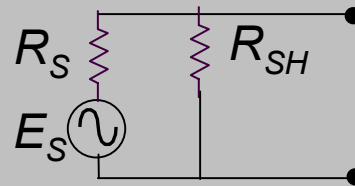


$$G_{SE} = \frac{S_O}{S_i} =$$

$$\frac{E_S^2 / 4(R_S + R_{SE})}{E_S^2 / 4R_S} = \frac{R_S}{R_S + R_{SE}}$$

$G_{SE}$  is greatest when  $R_S \gg R_{SE}$

### Gain of Shunt Resistor



$$G_{SH} = \frac{S_O}{S_i} =$$

$$\frac{(E_S \frac{R_{SH}}{R_S + R_{SH}})^2 / 4 \frac{R_S R_{SH}}{R_S + R_{SH}}}{E_S^2 / 4R_S} = \frac{R_{SH}}{R_S + R_{SH}}$$

$G_{SH}$  is greatest when  $R_S \ll R_{SH}$

*The gains of the resistor networks are less than one (1), and are often expressed instead as a power loss ratio,  $L = 1/G$ , which is then  $>1$ ; only  $G$  will be used here.*

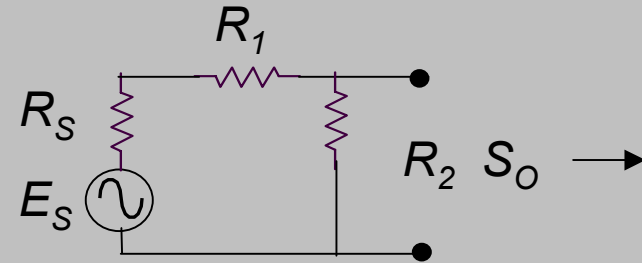


# Definition of Factors in Noise Figure (Cont'd)

## Available Gains Of Elementary Networks (Cont'd)

### Resistor L-Section Network

$$S_o = \left[ \frac{E_s R_2}{R_s + R_1 + R_2} \right]^2, S_i = \frac{E_s^2}{4 R_s}$$



$$G_L = \frac{S_o}{S_i} = \frac{R_s R_2}{(R_1 + R_s)(R_1 + R_2 + R_s)} = \left( \frac{R_s}{R_1 + R_s} \right) \left( \frac{R_2}{R_1 + R_2 + R_s} \right) = G_{SE} G_{SH}$$

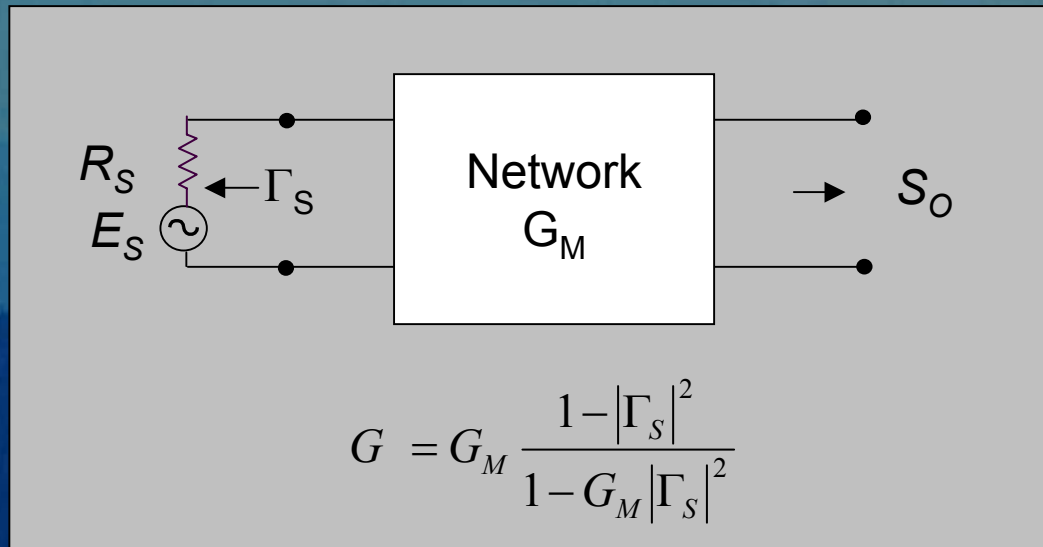
Gain is a maximum with  $R_s = \sqrt{R_1(R_1 + R_2)}$

*Available gain of L-Section is seen to be the product of the gains of the series section ( $R_1$  and  $R_s$ ) and the shunt section ( $R_2$  in parallel with series connection of  $R_s$  and  $R_1$ )*

# Definition of Factors in Noise Figure (Cont'd)

## Available Gains Of Elementary Networks (Cont'd)

### 2-Port Network



Where:

- $\Gamma_S$  is the reflection coefficient of  $R_S$  relative to the input characteristic impedance of the network
- $G_M$  is the maximum available gain of the network, i.e. the available gain with a matched input,  $\Gamma_S = 0$

# Definition of Factors in Noise Figure (Cont'd)

## Effective Noise Bandwidth, B

- Noise bandwidth, B, is defined as the equivalent rectangular pass band that passes the same amount of noise power as is passed in the usable receiver band, and that has the same peak in-band gain as the actual device has. It is the same as the integral of the gain of the device over the usable frequency band , i.e.:

$$B = \int_0^{\infty} \frac{G(f)}{G_0} df$$

Where:

- B is the effective noise bandwidth
- G(f) is the gain as a function of frequency over the usable frequency band
- G<sub>0</sub> is the peak value of in-band gain

*Typically, B is approximately equal to the 3 dB bandwidth.*

*For greatest sensitivity, B should be no greater than required for the information bandwidth.*

# Definition of Factors in Noise Figure (Cont'd)

## Available Input Noise

- Input noise,  $N_i$ , is defined as the thermal noise (Johnson Noise<sup>(2)</sup>) generated in the resistance of the signal source
- The input mean-square noise voltage is expressed concisely as<sup>(3)</sup>:

$$e_{ni}^2 = 4kT_oRB$$

Where

- $e_{ni}^2$  is the mean-square noise voltage of the Thevinin voltage source, in  $V^2/Hz$
- $k$  is Boltzman's constant =  $1.38 \times 10^{-23}$  Joules/°K
- $T$  is the absolute temperature of the resistor in °K
- In noise figure analysis, standard temperature is  $T_o = 290^\circ K$
- $R$  is the resistance, in Ohms
- $B$  is the effective noise bandwidth
- The available noise power,  $N_i$  (W), from the resistor is:

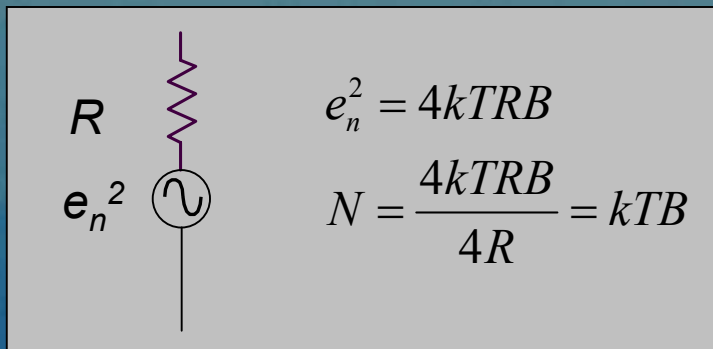
$$N_i = \frac{4kT_oRB}{4R} = kT_oB$$

- With  $T_o$  at  $290^\circ K$ ,  $N_i = 4.00 \times 10^{-21}$  W/Hz = -204.0 dBW/Hz = -174.0 dBm/Hz = -114.0 dBm/MHz

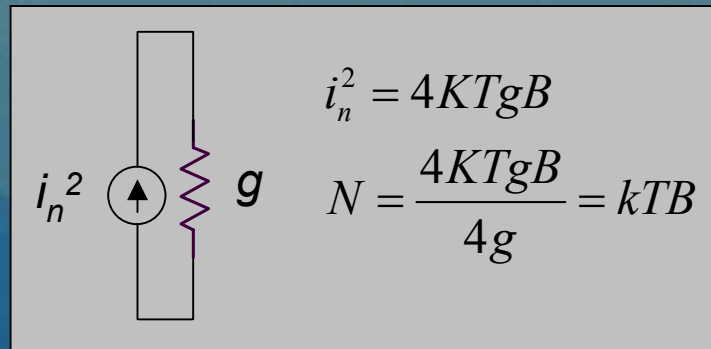
# Definition of Factors in Noise Figure (Cont'd)

## Thermal Noise Models (Cont'd)

### Thevinin Voltage Model



### Norton Current Model



- Where:

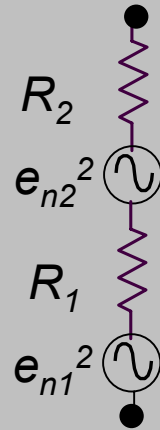
- $e_n^2$  is the mean-square noise voltage of the Thevinin voltage source, in  $V^2/\text{Hz}$
- $i_n^2$  is the mean-square noise current of the Norton current source, in  $A^2/\text{Hz}$
- $K$  is Boltzman's constant =  $1.38 \times 10^{-23}$  Joules/ $^\circ\text{K}$
- $T$  is the absolute temperature of the resistor in  $^\circ\text{K}$ 
  - In noise figure analysis, standard temperature is  $T_0 = 290^\circ\text{K}$
- $R$  is the value of the resistance, in Ohms
- $g$  is the value of the conductance, in mhos
- $B$  is the effective noise bandwidth, in Hz
- $N$  is the available noise power, in W

# Definition of Factors in Noise Figure (Cont'd)

## Thermal Noise Models (Cont'd)

Will network of resistors provide more noise power than  $kTB$ ?

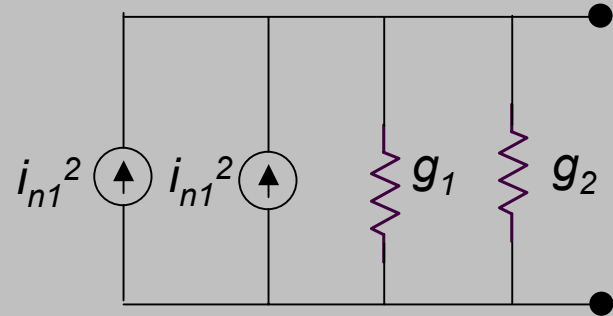
### Series Voltage Model



$$e_{n12}^2 = e_{n1}^2 + e_{n2}^2 = 4kTB(R_1 + R_2)$$

$$N_{12} = \frac{4kTB(R_1 + R_2)}{4(R_1 + R_2)} = kTB$$

### Shunt Current Model



$$i_{n12}^2 = i_{n1}^2 + i_{n2}^2 = 4kTB(g_1 + g_2)$$

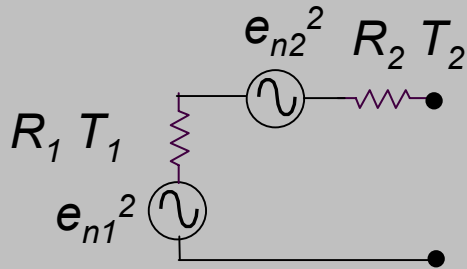
$$N_{12} = \frac{4kTB(g_1 + g_2)}{4(g_1 + g_2)} = kTB$$

*Available noise power from network of resistors at  $T$  is still just  $kTB$*

# Definition of Factors in Noise Figure (Cont'd)

## Thermal Noise Models (Cont'd)

### Series resistors at different temperatures



$$e_{n1-2}^2 = e_{n1}^2 + e_{n2}^2 = 4kB(R_1T_1 + R_2T_2) = 4kB[(R_1 + R_2)T_2 + R_1(T_1 - T_2)]$$

$$N_{1-2} = \frac{e_{n1-2}^2}{4(R_1 + R_2)} = kT_2B + k(T_1 - T_2)B \frac{R_1}{R_1 + R_2} =$$

$$kT_2B + G_{SE}k(T_1 - T_2)B$$

- When  $T_1 = T_2$ , the second term in  $N_{1-2}$  disappears and  $N_{1-2} = kT_2B$
- When  $T_1 \neq T_2$ , the term  $k(T_1 - T_2)B$  may be considered the excess available noise power of the source resistor,  $R_1$ .
- For noise calculations, the excess available noise power may be treated as though it was signal
- The excess noise power is then attenuated by the term  $R_1/(R_1 + R_2)$ , which is recognized as the gain of the series resistor, and the attenuated result adds to the  $kT_2B$ , power at the output\*

\*When  $T_1 < T_2$ , then  $N_{1-2} < kT_2B$

# Noise Figure Of Elementary Networks

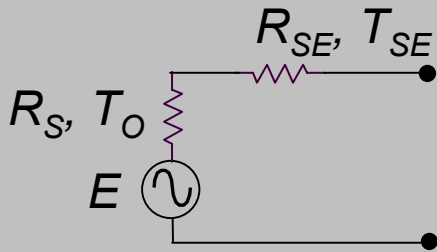
## Optimum Source Impedance for Minimum Noise Figure

- Each component in a receiver cascade can be characterized by an available Noise Figure (F) and available gain (G)
- The available noise figure of each component is dependent only on its source impedance within the receiver chain
- Every component having a noise figure has an optimum noise figure which is achieved when it is supplied from its optimum source impedance
- The optimum source impedance for components is not always the same as required for maximum gain, and so there will be an “optimum input mismatch”
  - In such cases, although the operational available output signal is reduced, the available output noise is reduced proportionally more



# Noise Figure Of Elementary Networks (Cont'd)

## F of Series Resistor



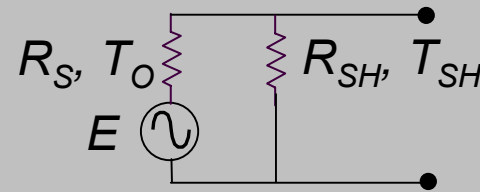
$$F = \frac{N_o}{GkT_oB} = \frac{kT_{SE}B + Gk(T_o - T_{SE})B}{GkT_oB}$$

$$= 1 + \frac{T_{SE}}{T_o} \left[ \frac{1}{G} - 1 \right]$$

Where,  $G = \frac{R_S}{R_S + R_{SE}}$

Noise figure is minimum when  $R_S \gg R_{SE}$

## F of Shunt Resistor



$$F = \frac{N_o}{GkT_oB} = \frac{kT_{SH}B + Gk(T_o - T_{SH})B}{GkT_oB}$$

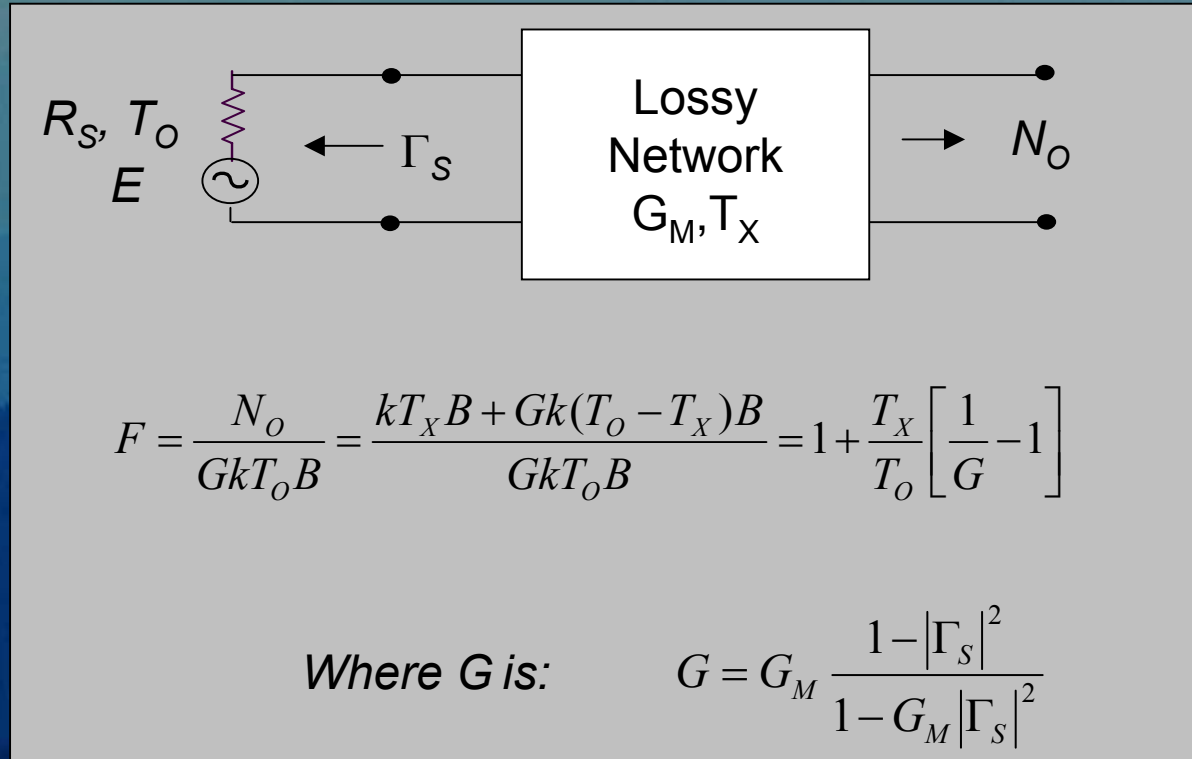
$$= 1 + \frac{T_{SH}}{T_o} \left[ \frac{1}{G} - 1 \right]$$

Where,  $G = \frac{R_{SH}}{R_S + R_{SH}}$

Noise figure is minimum when  $R_S \ll R_{SH}$

# Noise Figure Of Elementary Networks (Cont'd)

## Attenuators, Lossy Line Sections

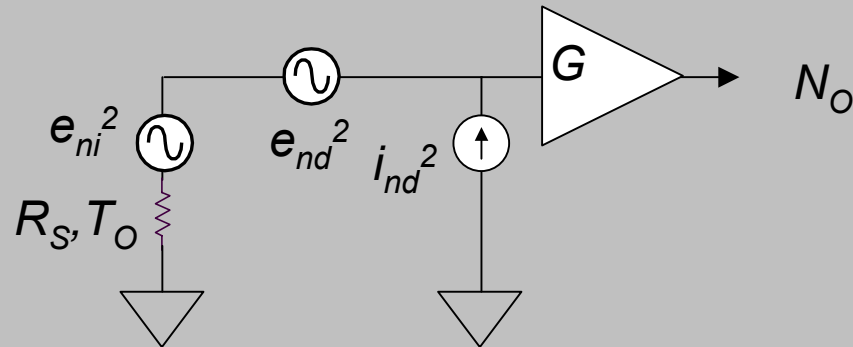


*By inspection,  $F$  is least when  $G$  is greatest ( $R_S$  is matched to the characteristic input impedance  $\Gamma_S = 0$ )*

# Noise Figure Of Elementary Networks (Cont'd)

## Active Devices

*Simplified noise model for active devices references all noise sources to input*



*Output noise power,  $N_{O_i}$ , referenced to input is:*

$$N_{O_i} = \frac{e_{ni}^2 + e_{nd}^2 + i_{nd}^2 R_S^2}{4R_S} = \frac{4kT_O R_S + e_{nd}^2 + i_{nd}^2 R_S^2}{4R_S}$$

$$N_i = kT_O$$

$$F = \frac{N_{O_i}}{N_i} = 1 + \frac{e_{nd}^2 + i_{nd}^2 R_S^2}{4kT_O R_S}$$

$$F \text{ is minimum when } R_S = \frac{e_{nd}}{i_{nd}} \text{ and is, } F = 1 + \frac{e_{nd}^2}{2kT_O R_S}$$

# Noise Figure Of Elementary Networks (Cont'd)

## Example, Active Device

Find optimum noise figure and required source impedance for amplifier with input voltage and current noise sources for the amplifier specified as:

$$e_n^2 = 1.6 \times 10^{-18} \text{ V}^2/\text{Hz} \text{ and,}$$

$$i_n^2 = 4 \times 10^{-23} \text{ A}^2/\text{Hz}$$

$$F \text{ is minimum when } R_S = \frac{e_{nd}}{i_{nd}} \text{ and is, } F = 1 + \frac{e_{nd}^2}{2kT_O R_S}$$

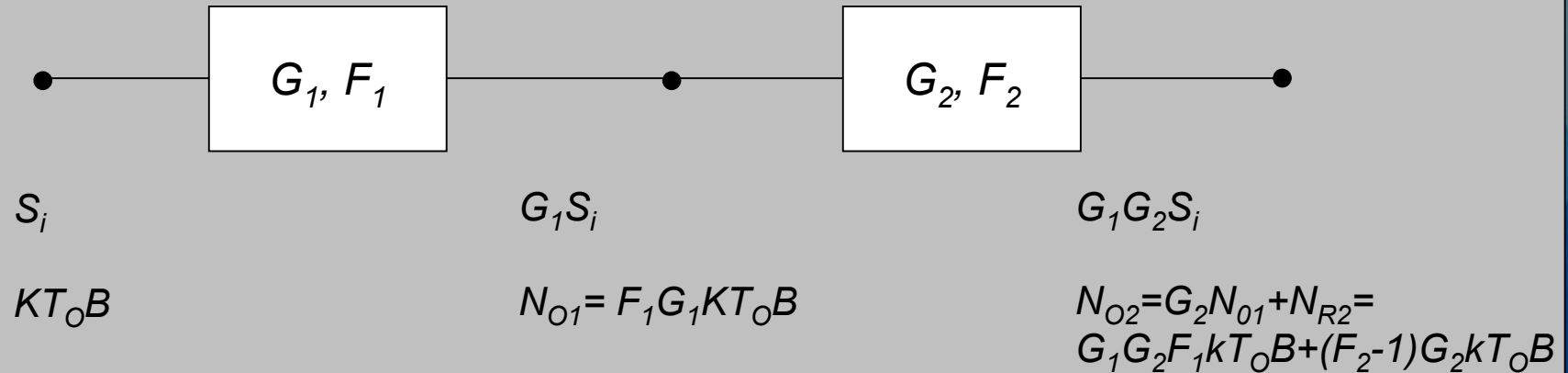
Therefore :

$$R_S = \sqrt{\frac{1.6 \times 10^{-18}}{4 \times 10^{-23}}} = 200 \text{ Ohms}$$

$$F = 1 + \frac{1.6 \times 10^{-18}}{2(1.38 \times 10^{-23})(290)(200)} = 2; \text{ NF} = 3 \text{ dB}$$

# Cascade Formula for Noise Figure

Noise Figure,  $F$  for cascade of 2 or more devices



$$F_{12} = \left( \frac{1}{G_1G_2} \right) \left( \frac{G_1G_2F_1kT_0B + (F_2 - 1)G_2kT_0B}{kT_0B} \right) = F_1 + \frac{F_2 - 1}{G_1}$$

In general, for cascade of  $n$  devices :

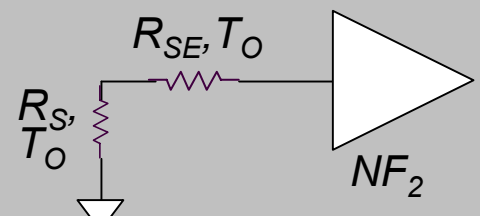
$$F_{1-n} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1G_2} + \dots + \frac{F_n - 1}{G_1G_2 \dots G_{n-1}}$$

# Cascade Formula for Noise Figure (Cont'd)

## Optimization Example

In the example on Slide 28, it was determined that the amplifier has an optimum noise figure,  $NF$ , of 3 dB when operated from a source impedance of 200 Ohms. When operating from a source of 50 Ohms, what is the best way to optimize for best noise figure? Using the cascade approach, the analyses below illustrate two optimization approaches. Which is best?

### Optimize with series 150 Ohm resistor

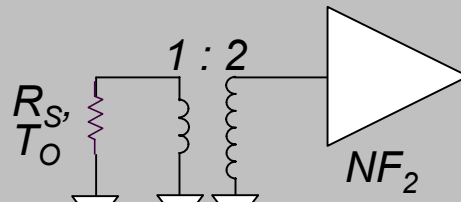


The diagram shows a source with impedance  $R_S$  and noise temperature  $T_0$  connected to a series resistor  $R_{SE}$  with noise temperature  $T_0$ . This combination is connected to an amplifier with noise figure  $NF_2$ .

$$F_{1-2} = F_1 + \frac{F_2 - 1}{G_1} = \frac{R_S + R_{SE}}{R_S} + \frac{F_2 - 1}{\left(\frac{R_S}{R_S + R_{SE}}\right)}$$

$$\frac{50 + 150}{50} + \frac{2 - 1}{\left(\frac{50}{50 + 150}\right)} = 8 = 9.03dB$$

### Optimize with input step-up transformer



The diagram shows a source with impedance  $R_S$  and noise temperature  $T_0$  connected to a 1:2 step-up transformer. The secondary of the transformer is connected to an amplifier with noise figure  $NF_2$ .

$$F_{1-2} = F_1 + \frac{F_2 - 1}{G_1} = 1 + \frac{2 - 1}{1} = 2 = 3dB$$

*Resistors in input matching networks adversely impact noise figure!*

# Cascade Formula for Noise Figure (Cont'd)

## Example, Receiver Cascade

<u>Component No.:</u>	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	Line Losses	BPF	Preamp	Line Losses	Mixer	IF Ampl	Rcvr
<i>G</i> (dB)	-0.1	-0.3	20	-1	-6	30	
<i>T</i> , <i>T<sub>e</sub></i> °K	290	320		320	380		
<i>F</i> (dB)			3			1	15
<i>G</i> (Ratio)	0.977	0.933	100	0.794	0.25	1000	-
<i>G</i> <sub>1-k</sub> (Ratio)	0.977	0.912	91.15	72.38	18.17	18166	
<i>F</i> * (Ratio)	1.026	1.079	2	1.286	4.906	1.259	31.62

$$F_{1-7} = 1.026 + \frac{1.079-1}{0.977} + \frac{2-1}{0.912} + \frac{1.286-1}{91.154} + \frac{4.906-1}{72.376} + \frac{1.259-1}{18.166} + \frac{31.62-1}{18166} =$$

$$1.026 + 0.081 + 1.096 + 0.003 + 0.054 + 0.014 + 0.002 = 2.278 = 3.576\text{dB}$$

(1)      (2)      (3)      (4)      (5)      (6)      (7)

\* For a resistive element,  $F = 1 + \frac{T_x}{T_o} \left( \frac{1}{G} - 1 \right)$

# System Noise Temperature Formulations

## Derivation

In low-noise systems and with low source temperature,  $T_S \ll T_O$ , treatment in terms of noise temperature<sup>(7)</sup>, rather than noise figure, is frequently preferred. Derivation follows.

- Output noise power level,  $N_O$ , for system operating from source at  $T_O$  is:

$$N_O = FGkT_O B = GkT_O B + (F-1)GkT_O B$$

- With a source temperature at  $T_S$ ,

$$N_O = GkT_S B + (F-1)GkT_O B = GkB(T_S + T_e)$$

- Where  $T_e = (F-1)T_O$  is defined as the effective input noise temperature of the receiver\*.

- The total equivalent noise temperature of the system referenced to its input terminals is  $T_{SYS} = T_S + T_e$ .

*\* $T_e$  is more concise than  $F$  in defining the noise performance of a receiver in that it is independent of source temperature.*



# System Noise Temperature Formulations, (Cont'd)

## System Sensitivity Analysis Using Noise Temperature

Signal source is antenna, pointed at sky with effective temperature,  $T_a=30^\circ\text{K}$ . Receiver system noise figure is  $\text{NF}=0.5\text{dB}$ ,  $F=1.122:1$ .

- The equivalent noise temperature of the receiver is:
  - $T_e=(1.22-1)290=35.4^\circ\text{K}$
- Therefore effective system noise temperature is:  
 $T_{\text{SYS}}=T_a+T_e=(30+35.4)=65.4^\circ\text{K}$
- Equivalent input system noise is\*:  
 $N_s = -174(\text{dBm/Hz}) + 10 \log(65.4/290) = -180.5 \text{ dBm/Hz}$

*\*The term:  $-174(\text{dBm/Hz})$  is the thermal level for  $290^\circ\text{K}$ ;  
the term:  $+10 \log(65.4/290)$ , corrects for a system noise temperature of  $65.4^\circ\text{K}$*

# System Noise Temperature Formulations (Cont'd)

## Cascade Formulation

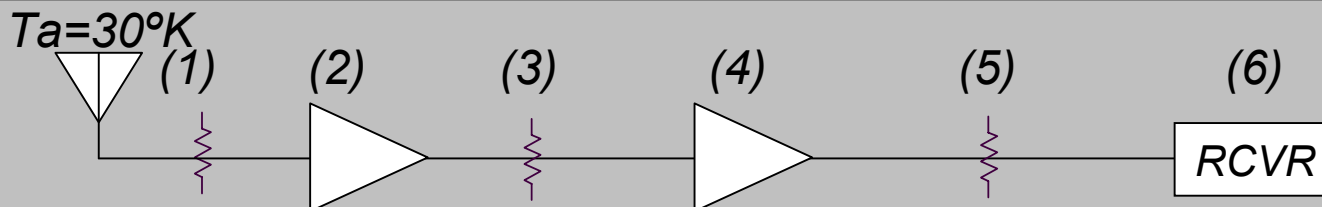
For cascade of n devices the noise temperature of the cascade is,

$$T_e = T_{e1-n} = (F_{1-n} - 1)T_O = \left( F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \dots + \frac{F_n - 1}{G_1 G_2 \dots G_{n-1}} - 1 \right) T_O$$
$$= T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 G_2} + \dots + \frac{T_n}{G_1 G_2 \dots G_{n-1}}$$

The Noise Temperature of the system, including the source at  $T_S$  is;  $T_{SYS} = T_S + T_e$

# System Noise Temperature Formulations (Cont'd)

## Example Receiver Analysis



	(1)	(2)	(3)	(4)	(5)	(6)
	Line Loss	Preamp	Line Loss	Postamp	Line Loss	RCVR
Gain (dB)	-0.2	14	-0.05	20	-0.5	-
Gain Ratio	0.955	25.119	0.989	100	0.891	-
F(dB)	-	-	-	-	-	3
T(°K)	320	-	320	-	320	-
Te(°K)*	15.081	35	3.705	125	39.046	290

$$T_e = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \frac{T_{e5}}{G_1 \dots G_4} + \frac{T_{e6}}{G_1 \dots G_5} =$$

$$15.081 + \frac{35}{0.955} + \frac{3.705}{0.955 \times 25.119} + \frac{125}{0.955 \times 25.119 \times 0.989} + \frac{39.046}{0.955 \times 25.119 \times 0.989 \times 100} +$$

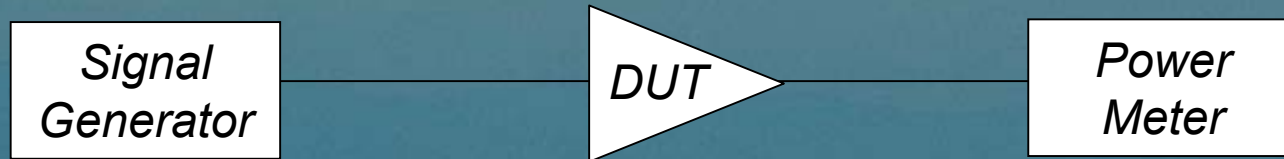
$$\frac{290}{0.955 \times 25.119 \times 0.989 \times 25.1 \times 100} = 15.081 + 36.649 + 0.154 + 5.269 + 0.017 + 0.131 = 57.309^\circ\text{K}$$

(1)      (2)      (3)      (4)      (5)      (6)

\* For a resistive element,  $T_e = T \left( \frac{1}{G} - 1 \right)$

# Measurement of Noise Figure

## Signal Generator Method<sup>(5,7)</sup>



### Procedure

- Tune Signal generator over frequency to measure output variation of power
- From data, determine B (Slide 22)
- Turn signal generator off, and note output noise power level,  $N_o = FGkT_oB$
- Turn signal generator on and tune to frequency of maximum G; adjust its level to  $S_i$  to just double output indication, to  $2 N_o$
- Then  $GS_i = FGkT_oB$ , and  $F = S_i / kT_oB$
- $NF(dB) = S_i(dBm) + 114 - 10 \log B(\text{MHz})$

### Example

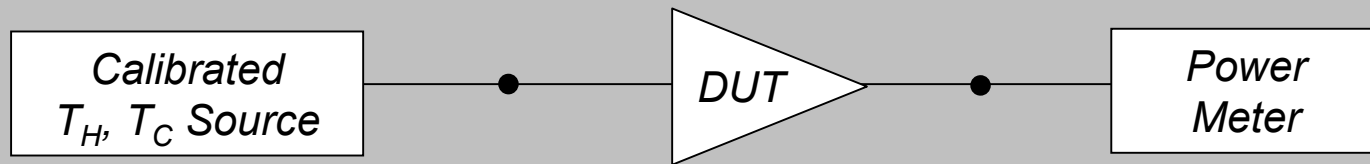
$B = 0.5 \text{ MHz}$ , (-3 dB MHz)

$S_i = -90 \text{ dBm}$

Therefore:  $NF(dB) = -90 + 114 - (-3) = 27 \text{ dB}$

# Measurement of Noise Figure (Cont'd)

## Calibrated Noise Source Method (Y Factor)<sup>(7)</sup>



$$kT_H B \longrightarrow$$

$$kT_C B \longrightarrow$$

$$N_{OH} = FGkT_O B + (T_H - T_O)kBG$$

$$N_{OC} = FGkT_O B + (T_C - T_O)kBG$$

$$\therefore Y = \frac{N_{OH}}{N_{OC}} = \frac{FT_O + T_H - T_O}{FT_O + T_C - T_O}; \quad F = \frac{\left(\frac{T_H}{T_O} - 1\right) - Y\left(\frac{T_C}{T_O} - 1\right)}{Y - 1}$$

### **Example:**

$T_H = 10,290^\circ\text{K}$  (argon source),  $T_C = 300^\circ\text{K}$

Measured Y factor:  $Y = 9 \text{ dB}$  (7.94:1)

Then,

$$F = \frac{\frac{10290}{290} - 1 - 7.94\left(\frac{300}{290} - 1\right)}{7.94 - 1} = 4.94; \quad NF(\text{dB}) = 10 \log(4.94) = 6.9 \text{ dB}$$

# Measurement of Noise Figure (Cont'd)

## Comparison of Measurement Methods

	<b>Signal Generator Method</b>	<b>Noise Source Method</b>
<b>Advantages</b>	<p>Accurate results even with significant out-of-band responses</p> <p>Useful for measurement of devices with high noise figure</p>	<p>Does not require separate determination of B</p> <p>Measurement is simple and straight-forward</p> <p>Lends itself to automatic measurements</p>
<b>Disadvantages</b>	<p>Requires separate measurement of B</p> <p>Not easily adaptable to automatic measurements</p>	<p>Capabilities of available noise sources don't allow measurement of devices with high noise figure</p> <p>Requires correction for presence of out-of-band responses</p>

# Presentation Summary

- The background and application of the concepts of Noise Figure and Noise Temperature for characterizing the fundamental limitations on the absolute sensitivity of receivers were set forth in a step-by-step approach and illustrated with examples to provide insight into the concepts.
- The origin of the major components of receiver noise, and their characteristics were summarized.
- Sample Noise Figure and Noise Temperature analyses of receiver systems were illustrated.
- The basic methods of measurement of Noise Figure and Noise Temperature were described and compared.
- References and a Bibliography follow. The Bibliography is intended to serve as a basis for further study.

# References

- (1) Friis, H.T., Noise Figures of Radio Receivers, Proc. Of the IRE, July, 1944, pp 419-422
- (2) Johnson, J. B. *Thermal Agitation of Electricity in Conductors*, Physical Review, July, 1928, pp 97-109
- (3) Nyquist, H. *Thermal Agitation of Electric Charge in Conductors*, Physical Review, July, 1928, pp. 110-113
- (4) Joshua Israelsohn. *Noise 101*, EDN January 8, 2004, PP. 41-48.
- (5) Stanford Goldman. *Frequency Analysis, Modulation and Noise*. McGraw-Hill Book Company, Inc., 1948
- (6) Van der Ziel, Aldert. *Noise: Sources, Characterization, Measurement*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970
- (7) D. C. Hogg and W. W. Mumford, The Effective Noise Temperature of the Sky, The Microwave Journal, March 1960, pp 80-84.



# Bibliography

## Agilent Application Notes

*The following 5 Agilent Application Notes are highly recommended for study and reference. Together, they provide excellent material on noise figure and noise temperature. They progress from the background of noise and receiver sensitivity, through a summary of computer-aided design of amplifiers for optimum noise figure, description of measurement setups and techniques, including analysis of measurement uncertainty. They include comprehensive lists of references. They can be ordered from the Agilent web site.*

- *Fundamentals of RF and Microwave Noise Figure Measurements, Application Note 57-1*
- *Noise Figure Measurement Accuracy- The Y-Factor Method, Application Note 57-2*
- *10 Hints for Making Successful Noise Figure Measurements, Application Note 57-3*
- *Noise Figure Measurement of Frequency Converting Devices, Using the Agilent NFA Series Noise Figure Analyzer, Application Note 1487*
- *Practical Noise-Figure Measurement and Analysis for Low-Noise Amplifier Designs, Application Note 1354.*

# Bibliography (Cont'd)

## Text Books

van der Ziel, Aldert. *Noise: Sources, Characterization, Measurement*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1970.-*Everything you ever wanted to know about noise but were afraid to ask. The classic text on the subject. Covers noise and noise models for devices, at a high technical level*

Stanford Goldman. *Frequency Analysis, Modulation and Noise*. McGraw-Hill Book Company, Inc., 1948. |

Norman Dye and Helge Granberg. *Radio Frequency Transistors, Principles and Applications*, EDN Series for Design Engineers, Motorola Series in Solid State Electronics. *Contains examples of noise figure analysis models of amplifier stages*

# Bibliography (Cont'd)

## Background

Johnson, J. B. *Thermal Agitation of Electricity in Conductors*, Physical Review, July, 1928, pp 97-109. *This is the classic paper on measured thermal noise in resistors. It showed that the mean square thermal noise voltage at the terminals of a resistor were proportional to the magnitude of the Ohmic value of its resistance and to its absolute temperature. The noise voltage is referred to as “Johnson noise” in his honor.*

•Nyquist, H. *Thermal Agitation of Electric Charge in Conductors*, Physical Review, July, 1928, pp. 110-113. *This is the classic paper companion paper to the Johnson Paper which related the Johnson noise to the fundamental laws of thermodynamics and arrived at the famous, KTB*

•Friis, H.T., *Noise Figures of Radio Receivers*, Proc. Of the IRE, July, 1944, pp 419-422. *This is the classic paper on Noise Figure. It should be reviewed as an excellent example of a concise presentation of an important concept.*

# Bibliography (Cont'd)

## Background (Cont'd)

Harold Goldberg. *Some Notes on Noise Figure*, Proc. Of the IRE, October, 1948, pp 1205-1214. *The paper is a tutorial based on the Friis paper. It carefully explores the basics concepts of noise figure and illustrates their application, progressing from elementary networks to low-noise vacuum tube amplifier configurations. The paper's approach served as a model for this presentation.*

Kurt Stern. *"Fundamentals of Electrical Noise"*, Presentation to the IEEE MTT Section of Long Island NY, January, 2004. *This reviews the basics of noise figure and its measurement. It includes detail of automatic measurement setups and of available test equipment. The presentation is available on the web site for the LI NY MTT Chapter of the IEEE*

Joshua Israelsohn. *Noise 101*, EDN January 8, 2004, PP. 41-48. *Good summary on thermal, shot, and jitter noise.*